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Abstract

Under imperfect competition, firm's incentive to deviate from a collusive agreement is usually short term: the firm increases its immediate profit but reduces its future profits due to the upcoming competitors' reaction. When firms have to invest to increase their production, and that investment is irreversible, a firm may deviate to preempt its competitors and obtain a dominant position on the market. When firms are more patient, preemption is more profitable, and collusion may thus be harder to sustain. This result, contrasting with the literature on collusion, suggests that folk theorems may be inappropriate to study collusion in Markovian frameworks.

Keywords: Capacity investment and disinvestment. Dynamic games. Markov perfect equilibrium. Real options. Collusion.

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Contents

1 Introduction

In many industries, the production of firms is limited by their infrastructure, i.e. the amount of factories or equipment that they own. This production capacity may evolve in time, as firms invest in new infrastructure. However, the cost of building a new capacity is usually sunk and investments are thus (at least partially) irreversible. These features of capacity limitation and irreversible investment impact the way competition works. This is the case in a famous example of collusion: the lysine cartel.

Lysine is an essential amino acid that stimulates growth and lean muscle development in hogs, poultry, and fish. It has no substitute. By the late 1960s, Japanese biotechnology firms had discovered a bacterial fermentation technique that transformed the production of lysine. The process involves the fermentation of dextrose into lysine and requires a specific chemical infrastructure. The cost of this infrastructure is therefore sunk.[1] This technique is considerably cheaper than conventional extraction methods. By the end of the 1980s, there were three major players in the lysine market: Ajinomoto and Kyowa Hakko based in Japan, and Sewon based in South Korea. In 1988, ADM acquired a fermentation technique for lysine and began building the world's largest lysine factory in 1989. ADM's entry was effective in February 1991.

This entry of ADM into the lysine market led to the creation of one of the best-known cartels in history. The consortium ran from June 1992 to June 1995, date at which the FBI raided the headquarters of the participating firms. It was the first global price-fixing conspiracy to be convicted by US or EU antitrust authorities in 40 years, and the financial penalties amounted to $305 million. The cartel started just after Archer-Daniels-Midland Company (ADM) entered into the lysine business. However, between the end of 1992 and early 1993, ADM invested massively to increase its production capacity, which rose from 60 000 tons per year to 113 000 tons. At the same time, ADM started to cheat on the collusive agreement, and a price war began in March 1993. The price war ended in November 1993, and the cartel carried on peacefully until the intervention of the FBI.[2]

[1] There is some possibility of transforming a lysine plant to produce another amino acid, however this is costly.

[2] For more information on the lysine cartel, see the books by Eichenwald (2000) and Connor (2000), and

This cartel case emphasizes the following point: capacities can evolve during periods of collusion. Several papers focus on the impact of constant capacity constraint on collusion, but very few deal with the strategic evolution of capacity. This work attempts to fill that gap.

More precisely, this work shows the existence of long-run incentives for deviation. Indeed, to deviate from a collusive equilibrium, a firm has to invest in new units of capacity in order to increase its production. In doing so, the deviating firm commits to its new capacity level due to the irreversibility of investment. This reduces its opponent's incentive to invest in order to punish the deviating firm. In some cases, it even prevents the opponent from implementing any punishments. A firm may thus increase its long-run profit by deviating from the collusive equilibrium. This contrasts with the standard theory of collusion, whereby deviation's only effect is to increase firm's short-run profits, while reducing long run profits due to punishment. This is consistent with the lysine case, where ADM's deviation in 1993 and the building of new capacity allowed the firm to increase its market share in the American market from 44 percent in 1992 to 57 percent in following years.

In the literature on collusion and capacity constraint, Compte et al (2002) focus on the case of price competition with inelastic demand under fixed capacity constraints, and show that larger firms have the most incentives to deviate. Under Cournot competition, linear demand and soft capacity constraint, Vasconcelos (2005) finds that the smallest firms have the most incentives to deviate. Under a similar framework, Bos and Harrington (2010) show that, when a cartel is not inclusive, deviation may be driven by medium-sized firms. In all of these works, asymmetry between firms' capacity hinders collusion.

Fabra (2006) and Knittel and Lepore (2010) focus on the impact of capacity constraint on collusion in presence of demand evolution, and show that a collusive price can be counter-cyclical. Garrod and Olczak (2014) show that, under imperfect monitoring, capacity constraints may help to detect a deviation.

All of these works assume that capacities are fixed and study the impact of capacities on collusion.[3] To my knowledge, only two papers focus on the evolution of capacity during

the articles by Connor (2001), Roos (2006) and Connor (2014).

[3] Compte et al (2002) and Vasconcelos (2005) study the impact of a change in the capacity distribution due to a merger, and Knittel and Lepore (2010) endogenize the choice of capacity at the beginning of the

collusion. Paha (2016) adapts the model devised by Besanko and Doraszelski (2004) to simulate the investment behavior of firms when they collude in price but not in capacity. Besides his result on the effect of uncertainty on cartel formation, Paha presents counter-intuitive evidence that a low discount rate can impair collusion. In a duopoly model with linear demand and partially irreversible investment, Feuerstein and Gersbach (2005) studies a grim trigger strategy in which the cooperative behavior for each firm involves installing half of the monopoly capacity, and the punitive behavior involves investing until the marginal value of capacity equals the price of investment. They show that these strategies can form an equilibrium, but at a lower discount rate compared to the case of fully reversible investment.

In the present paper, I study a Cournot duopoly with irreversible investment in capacity in a discrete time setting. More precisely, at each point in time, the production of each firm is determined by its level of capacity. Firms may decide to increase their capacity through buying assets. The price of investment is linear. The results depend on the period of time, which can be interpreted as the degree of flexibility of the firms' investment decisions. When the period is long, it takes time for the firm to punish a deviation, whereas when the period is close to zero, firms react instantly to any deviation. This model can thus be viewed as a generalization of Feuerstein and Gersbach (2005).

In this framework, firms invest only at the beginning of the game in the non-cooperative equilibrium. If both firms start with sufficiently low initial capacity, then they invest to the same Cournot level of capacity. When one of the firms starts out with an initial capacity that is higher than the Cournot level, then the firm is committed to this level of capacity due to the irreversibility of investment, and thus its opponent reacts by investing at a capacity level lower than the Cournot level. The firm with the high initial capacity does not invest.

To study collusion, I focus on a particular kind of strategy, i.e. the grim-trigger strategy for status quo. This consists in the firm keeping its initial capacity as long as its opponent also keeps its own initial level of capacity. When one of the firms deviates, the other responds by investing as in the non-cooperative equilibrium. However, if the deviating firm has installed a capacity larger than the Cournot outcome, then it has committed itself to this capacity for the rest of the game, and the non-cooperative equilibrium is for the punishing firm to invest less than the Cournot outcome. This is the long run effect of the deviation: the deviating

game. However, at the time when collusion is implemented, the capacities are fixed.

firm can gain a size advantage during the deviation, which persists for the rest of the game. The usual short-run effect is also present: there is a period, between the implementation of the deviation and the implementation of the punishment, when the deviating firm is the only firm to have a capacity larger than the collusive capacities. Both short-run and long-run effects are important to determine a firm's incentives to deviate.

Focusing on the limit case in which the period of time shrinks to zero allow to isolate the long run effect. In such case, there is a parallel between the decision of the Stackelberg leader and the optimal deviation. Indeed, the punishing firm decides on its capacity taking into account the capacity already installed by the deviating firm. Collusion is then only possible if the Stackelberg profit is under half of the monopoly profit. A low discount rate is thus harmful for collusion, as more patient firms are more likely to invest, and then prefer becoming the Stackelberg leader rather than obtaining half of the monopoly profit. When the period of time is positive, the short-run effect may compensate the long-run one and the impact of the discount rate on collusion will depend on which effect dominates.

These findings provide intuition on the results of Paha (2016) and Feuerstein and Gersbach (2005). In Paha (2016), the counter-intuitive effect of a low discount rate on the possibility of collusion comes from the domination of the long-run effect over the short-run effect. Feuerstein and Gersbach (2005) finds the opposite result (on discount rate) because the Stackelberg leader's profit with linear demand equals half of the monopoly profit. Therefore, the long-run effect does not permit collusion to be implemented (with linear demand), and collusion is more difficult with irreversible investment than with reversible investment as the short-run effect must compensate the negative long-run effect. This depends on the form of profit assumed; the introduction of a simple quadratic production cost leads to a different result.

Section 2 presents the model and the non-cooperative equilibrium. Section 3 focuses on the collusive equilibrium for status quo, with the short-run and long-run effects of deviation. Section 4 concludes.

2 A Framework with Irreversible investment

Model

I consider a market with two firms (A and B) competing *à la Cournot* with a homogenous product, in a dynamic time setting similar to that of Sannikov and Skrzypacz (2007). Time is continuous and the horizon is infinite, but firms only take decision at discrete times $t = \{0, \tau, 2\tau, ...\}$. The game is therefore discrete.[4] The continuous time framework makes it possible to vary τ, the duration of time periods between which investment decisions are made.

More precisely, at each time $t = n\tau$ ($n \in \mathbb{N}$) firms simultaneously decide whether extend their capacity through buying assets. Firms start with an initial capacity k_0^i (for $i \in \{A, B\}$) and the capacity of firm i at time t is thus $k_t^i = k_{t-\tau}^i + I_t^i$, where I_t^i denotes the investment of firm i at time t ($I_t^i \geq 0$). During the time interval $[t, t+\tau[$ the quantity produced by firm i is determined by its capacity leading to a production constraint:

$$\text{for all } s \in [t, t+\tau[,\ q_s^i = k_t^i. \tag{1}$$

The price of the good is a function of the total quantity produced, $P\left(q_t^A + q_t^B\right)$ and the production cost of firm i is $c(q_t^i)$. The price of investment, p^+, is linear, and firms face the same discount factor $\delta \in]0, 1[$. The profit of firm i made during the time interval $[t, t+\tau[$ is then fully determined by the firms' capacity at time t:

$$\int_t^{t+\tau} \delta^s \left[P\left(q_s^i + q_s^{-i}\right) q_s^i - c(q^i)\ \right] ds = \delta^t \frac{(1-\delta^\tau)}{\ln(1/\delta)} \left[P\left(k_t^i + k_t^j\right) k_t^i - c(k_t^i)\ \right] \tag{2}$$

where k_t^j is the capacity of the opponent of firm i. For a vector of capacity $k_t = \left(k_t^i, k_t^j\right)$, the inter-temporal profit of firm i is then given by:

$$\Pi_i = \sum_{t=0,\tau,..} \delta^t \left[(1-\delta^\tau)\, \pi_i(k_t) - p^+ I_t^i\right], \tag{3}$$

[4] Even if firms' decisions are taken at a discrete date, this game is not a repeated game stricto sensu, i.e. the stage games are not independent as the capacity installed at time t is present at time $t+\tau$.

where $\pi_i(k_t)$ is defined by:

$$\pi_i(k_t) = \frac{P\left(k_t^i + k_t^{-i}\right)k_t^i - c(k_t^i)}{\ln(1/\delta)}. \tag{4}$$

π_i is the inter-temporal profit of firm i when there is no investment during the game.

I assume that firms' strategies are Markovian, meaning that the investment at time t is a function of the capacities of the industry at time $t-\tau$, $I_t^i = \hat{I}^i(k_{t-\tau})$.[5] The profit of firm i can then be rewritten:

$$\Pi_i(k) = (1-\delta^\tau)\,\pi_i\left(k + \hat{I}(k)\right) - p^+ I^i(k) + \delta^\tau \Pi_i\left(k + \hat{I}(k)\right). \tag{5}$$

$\hat{I}^*(k)$ is then a Markovian equilibrium if and only if, for each $i \in \{A, B\}$,

$$\hat{I}^{*i}(k) = \arg\max_{I^i}\left\{(1-\delta^\tau)\,\pi_i\left(k^i + I^i,\ k^j + \hat{I}^{*j}(k)\right) - p^+ I^i + \delta^\tau \Pi_i\left(k^i + I^i,\ k^j + \hat{I}^{*j}(k)\right)\right\}, \tag{6}$$

where $\hat{I}^{*j}(k)$ is the strategy of firm j, and the equilibrium profits, Π^*, are defined by:

$$\Pi_i^*(k) = (1-\delta^\tau)\,\pi_i(k + \hat{I}^*(k)\,) - p^+ \hat{I}^i(k) + \delta^\tau \Pi_i^*\left(k + \hat{I}^*(k)\right). \tag{7}$$

The following assumption ensures the existence of Markovian equilibria.

Assumption A: The cost function, $c(.)$, is a twice-differentiable positive function such that $c' \geq 0$, $c'' \geq 0$. $P(.)$ is also a twice-differentiable positive function, with $P' < 0$, $P'' < 0$ when P is strictly positive.

Non-cooperative Equilibrium

As usual in dynamic competition with an infinite horizon, there exist a multiplicity of equilibria. The issue is to determine which equilibrium is the non-cooperative one. In a classic model of repeated Cournot competition, the non-cooperative equilibrium is given by the repetition of stage game equilibrium. However, in the present model, periods are linked

[5] For simplicity of notation, a variable with a hat will be a function of the state variable, a variable with a capital index will indicate a function of the competitor's capacity.

by firms' capacity, and the stage game depends on the past. There is then no clear non-cooperative equilibrium.[6] The objective of this sub-section is to defined one of the Markovian equilibrium as the non-cooperative one.

There exists a Markov perfect equilibrium in which the firms' strategies are to invest in the first period, and not in the rest of the game. This Markov perfect equilibrium is defined as the non-cooperative one. There are three reasons for this. First, the firm's strategies are simple, without any punishment scheme (as the firm does not invest after the first period). Second, when firms start with the same level of capacity, this equilibrium corresponds to the classic equilibrium of repeated Cournot competition. Finally, let us define a finite game with the same payoff functions but in which firms can only take decisions for the first T period of the game ($T > 1$). This game features a unique Markov perfect equilibrium that exhibits the same investment behavior as the equilibrium defined to be the non cooperative one.[7]

In order to state proposition 1, let k^i_{BR} be the best response capacity of firm i, i.e. the level of capacity that firm i will install if firm j installs a level k^j, and firm i has no initial capacity:

$$\frac{\partial \pi_i}{\partial k^i}(k^i_{BR}, k^j) = p^+. \tag{8}$$

Indeed, when there is no investment, $\Pi_i = \pi_i$. As the profits are symmetric for both firm, $k^A_{BR}(.) = k^B_{BR}(.)$ and the best response capacity will be written $k_{BR}(.)$ in the following. The Cournot level, k_c, is then given by:

$$k_{BR}(k_c) = k_c. \tag{9}$$

The Cournot level is the equilibrium of the game when both firms have no initial capacities. When firms do have initial capacities, the equilibrium is given by proposition 1.

Proposition 1 *The following strategies define a Markov perfect equilibrium:*

$$\hat{I}^{*i}(k) = \max\left\{\hat{k}^{*i}(k) - k^i,\ 0\right\}, \tag{10}$$

[6]Unlike in standard stationary games, here there exists an infinity of stationary states, and the equilibrium depends on the path chosen by the firms, and on their initial capacities.

[7]This is also the unique sub-game perfect equilibrium.

where $\hat{k}^{*i}(k)$ *is defined by:*

$$\hat{k}^{*i}(k)=\begin{cases} k_{BR}(k^j) \text{ if } k^j > k_c \\ k_c \text{ if } k^j \le k_c \end{cases}. \tag{11}$$

All investments are made at the beginning of the game, and the equilibrium path is $\left\{\hat{I}^*(k_0),0,0,..\right\}$. *This equilibrium will be defined as the non-cooperative one.*

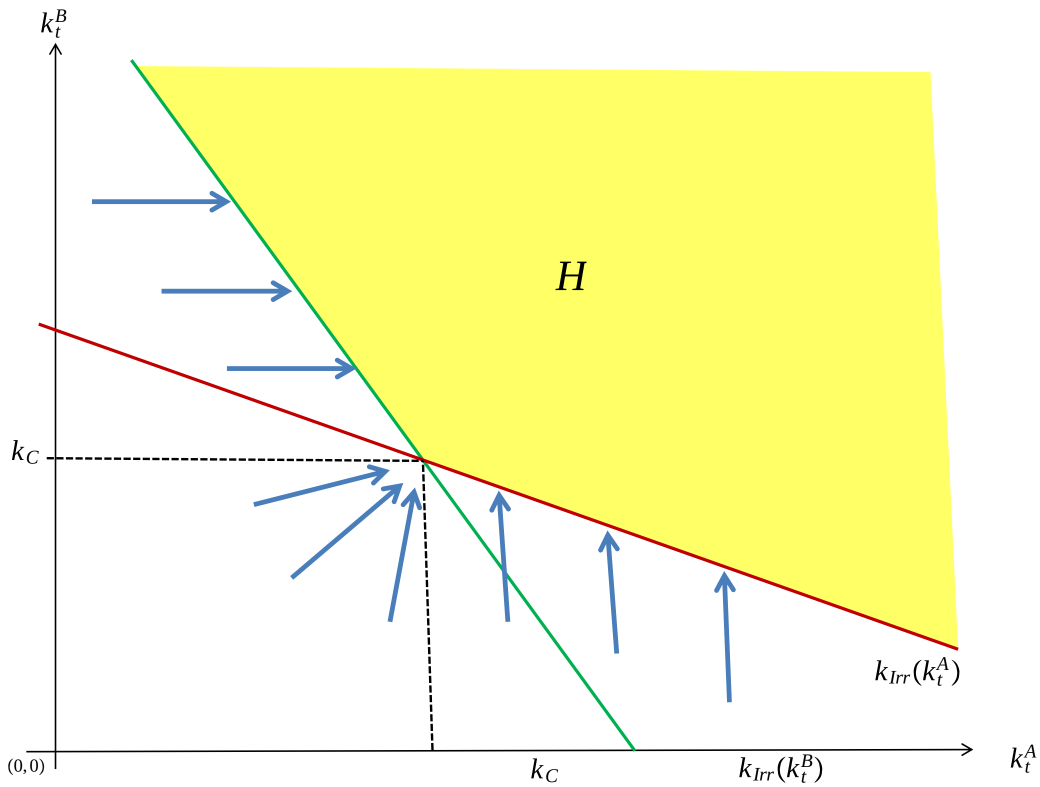

Figure 1: non-cooperative equilibrium

Proposition 1 presents the three possible scenarios depending on the initial capacities. If initial capacities are low enough, both firms invest, and the equilibrium is the Cournot equilibrium.[8] If one of the firms has a high initial capacity and the other firm a low one, the biggest firm will want to reduce its capacity but wil be constrained by the irreversibility of

[8]With a marginal cost of investment p^+.

capacity. The smaller firm then adjusts its capacity to the initial capacity of its opponent, investing until its best response level. Finally, when the initial capacity of both firms is too high, no one invests and the firms keep their initial capacities forever.

3 Collusive agreement for status quo

Agreement for status quo

In this section I consider a specific collusive strategy, i.e. the collusive agreement for status quo. This consists in each firm keeping its initial capacity as long as the other firm also keeps its initial capacity unchanged, and in investing as in the non-cooperative equilibrium if the other firm has invested. There are several reasons for using this particular grim-trigger strategy. In real cartel agreements, firms usually agree on quantities proportional to the capacity owned at the time of the agreement. While the grim-trigger strategy is the most considered strategy in the literature, some recent studies focus on other equilibria with renegotiation. However, in this case, the irreversibility of investment prohibits any renegotiation after the punishment, as the firms cannot reduce their investment (or their production).

More precisely, the grim-trigger strategy for status quo is defined by:

$$\hat{I}_{coll}(k) = \begin{cases} 0 \text{ if } k = k_0 \\ \hat{I}^*(k) \text{ elsewhere} \end{cases} . \tag{12}$$

The objective of this section is to characterize the set of initial capacities such that the grim-trigger strategies (12) form a Markovian equilibrium:

$$\Psi = \left\{ k_0 \in \mathbb{R}_+^2 \mid I_{coll} \text{ verifies (6) and (7)} \right\} . \tag{13}$$

I assume in the following that the initial capacities are sufficiently low for both firms to invest in the non-cooperative equilibrium (as defined in the previous section). If both capacities are high and no firms invest at the non-cooperative equilibrium, then the agreement for status quo is obviously an equilibrium, as it coincides with the non-cooperative equilibrium.

In that case, there is no real collusion, as firms behave exactly in the same way in both equilibria. If one of the firms has a capacity higher than the Cournot capacity, given by (9), and the other firm has a low enough capacity to invest in the non-cooperative equilibrium, then there is no possibility for a status quo agreement. Indeed, when the largest firm keeps its capacity constant, the smallest firm's best response is to invest up to the capacity level (8). This corresponds to the non-cooperative equilibrium behavior, and for the smallest firm, the best response to the status quo agreement is to deviate from the collusive agreement.

Assume that firm i deviates from the agreement for status quo by investing in a level of capacity I_d. At the time of the investment decision, its competitor does not know that firm i is deviating and therefore keeps its initial capacity. When the investment is made, the opponent observes the deviation and the non-cooperative equilibrium is played out. However, at this point in time, firm i's capacity is no longer its initial capacity, but its capacity of deviation, $k_d^i = k_0^i + I_d$. The non-cooperative equilibrium played after the deviation may be impacted by this evolution of capacity.

The form of the non-cooperative equilibrium implies two properties of the optimal deviation.

If the capacity of deviation is below the Cournot level ($k_d^i < k_c$), then for both firms the non-cooperative equilibrium is to invest until the Cournot level. In that case, by investing directly up to the Cournot level, firm i does not change the non-cooperative equilibrium established after carrying out the deviation, and it increases its profits during the time when the deviation capacity is installed, but not the punishment capacity. The optimal deviation should then be higher than the Cournot level. This also implies that the deviating firm i does not invest after the deviation.

Furthermore, there exists a level of capacity, $(k_{BR})^{-1}(k_0^j)$, such that, if the deviating capacity is greater than this level of capacity, the punishing firm does not invest after the deviation. Indeed, due to the irreversibility of investment, the deviating firm is committed to a capacity sufficiently high to prohibit the other firm from investing at the non-cooperative equilibrium. If the deviating capacity is greater than this level, then reducing the deviating capacity does not change the reaction of the punishing firm, but enables greater profits for the deviating firm.[9] The optimal deviation should then be lower than this level of deviation.

[9]Indeed, when the capacity of firm i is $(k_{BR})^{-1}(k_0^j)$ and the capacity of firm j is k_0^j the non-cooperative

These features are summarized in the following Lemma.

Lemma 1 *The optimal deviation verifies* $k_d^{*i} \in \left[k_c, (k_{BR})^{-1}\left(k_0^j\right)\right]$.

In the following, I focus on deviations which verify the condition given in Lemma 1, in order to find the optimal deviation. In that case, the profit of firm i is:

$$\Pi^i = (1-\delta^\tau)\,\pi_i(k_d^i, k_0^j) - p^+\left(k_d^i - k_0^i\right) + \delta^\tau \pi_i\left(k_d^i, k_{BR}\left(k_d^i\right)\right). \tag{14}$$

The first term of the profit is obtained when the deviating firm has installed its capacity, but before the punishment is implemented. The second term is simply the investment cost of the deviation. The last term occurs during the rest of game, when firms reach the non-cooperative equilibrium. It depends on the capacity installed during the deviation. The deviation therefore has two impacts: it increases the short-run profit and it also reduces the capacity installed by the competitors in the long-run, during the punishment phase.[10]

Infinitely short time period

This sub-section focuses on a case where the time period goes to zero ($\tau \to 0$). As time is continuous, the deviation is instantly detected and both the deviation capacity and the punishment capacity are instantly installed. The deviating firm does not make any profit before the implementation of its capacity or during the time between the deviation and the punishment. Indeed, when $\tau \to 0$, the deviating firm's profit becomes:

$$\Pi_i = \pi_i(k_d^i, k_{BR}\left(k_d^i\right)) - p^+\left(k_d^i - k_0^i\right). \tag{15}$$

Therefore, deviating from the collusive equilibrium has no short-run impact. It only influences the long-run distribution of capacity.

In this limit case, there is a clear parallel between the choice of the optimal deviation and the Stackelberg game. Due to the irreversibility of investment, when a firm deviates, it

equilibrium that no firm invests. Firm i has then no interest to invest more than $(k_{BR})^{-1}\left(k_0^j\right)$.

[10] Assumption A implies that the capacity installed in punishment, determined in (8) is decreasing.

commits to its new level of capacity. When the opponent observes the deviation, it reacts to this new choice of capacity. After this point, firms maintain their capacity forever. As the time period goes to zero, firms only make a profit in the long run. The deviating firm is then in the position of a Stackelberg leader, whereas its opponent, which reacts to the deviation, behaves as a Stackelberg follower.

Let k_s^i be the Stackelberg capacity, as defined by the first order condition of (15):

$$\frac{\partial \pi_i}{\partial k^i}(k_s^i, k_{BR}\left(k_s^i\right)) + \frac{\partial k_{BR}}{\partial k^i}\left(k_s^i\right)\frac{\partial \pi_i}{\partial k^j}(k_s^i, k_{BR}\left(k_s^i\right)) = p^+. \tag{16}$$

Lemma 2 describes the optimal deviation.

Lemma 2 *When $\tau \to 0$, the optimal deviation is to install a capacity:*

$$\hat{k}_d^{*i} = \begin{cases} k_s^i \text{ if } k_0^j \leq k_{BR}\left(k_s^i\right) \\ \left(k_{BR}\right)^{-1}\left(k_0^j\right) \text{ if } k_0^j > k_{BR}\left(k_s^i\right) \end{cases}. \tag{17}$$

If the punishing firm starts with a low initial capacity, the punishing firm will invest after the deviation. Knowing that its opponent will invest after the deviation, the optimal strategy of the deviating firm is to install the Stackelberg capacity. In such case, the condition for firm i to accept the collusive agreement is to make more profits with its initial capacity than if it installs the Stackelberg capacity and its opponent invests to the best response level of the Stackelberg capacity:

If $k_0^j \leq k_{BR}\left(k_s^i\right)$,

$$\pi_i(k_0) \geq \pi_i(k_s^i, k_{BR}\left(k_s^i\right)) - p^+\left(k_s^i - k_0^i\right). \tag{18}$$

When the initial capacity of the punishing firm is sufficiently high, the deviating firm invests less than the Stackelberg capacity in order to take into account the fact that the punishing firm is committed by its initial level of capacity. In such case, the punishing firm does not invest after the deviation. The condition for firm i to accept the collusive agreement is thus to make more profit with its initial capacity than if it increases its capacity to the best response level:

If $k_0^j > k_{BR}\left(k_s^i\right)$,

$$\pi_i(k_0) \geq \pi_i\left(\left(k_{BR}\right)^{-1}\left(k_0^j\right), k_0^j\right) - p^+\left(\left(k_{BR}\right)^{-1}\left(k_0^j\right) - k_0^i\right). \tag{19}$$

Figure B presents the optimal deviation and the punishment which follows in function of the initial capacities.

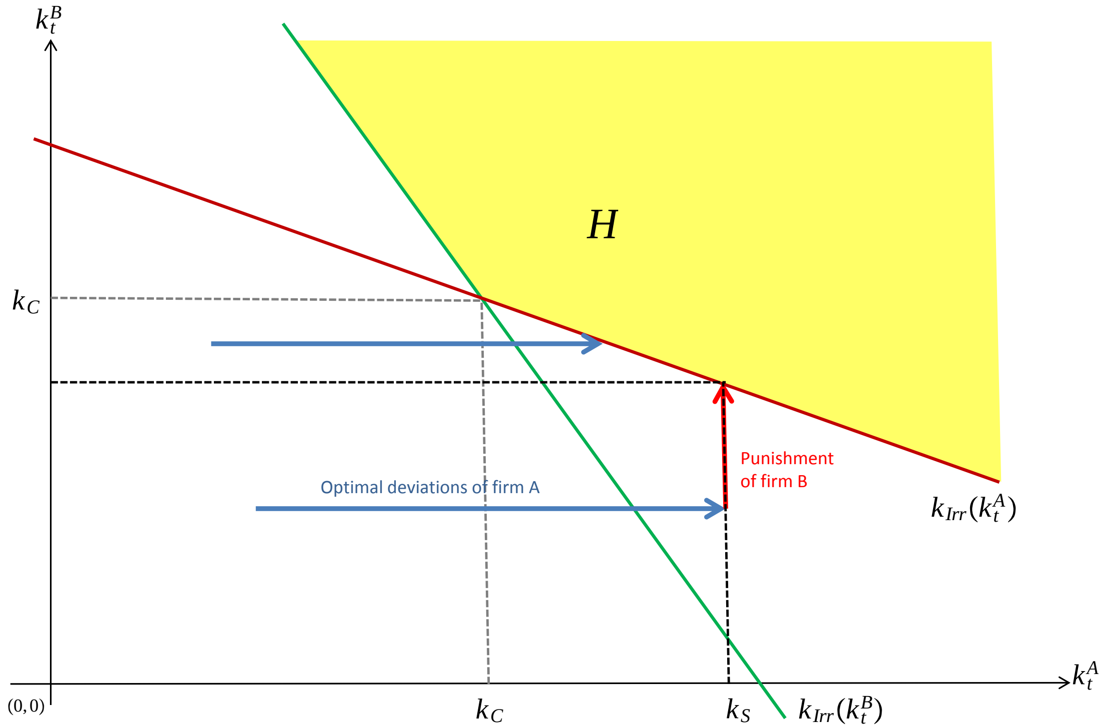

Figure 2: optimal deviation

The conditions (18) and (19) allow us to determine which firm has the more incentives to deviate from the collusive agreement.

Lemma 3 *When $\tau \to 0$, the firm with the most incentives to deviate from the collusive agreement is the firm with the lowest initial capacity:*

If (18) and (19) holds for i and $k_0^i < k_0^j$, then (18) and (19) holds for firm j.

Assume that both firms are small, so that the deviation involves installing the Stackelberg capacity, even when the larger firm deviates ($k_0^i \leq k_{BR}(k_s^i)$ for both $i = A, B$). In such case, the total capacity after the deviation is the sum of the leader and the follower capacity of Stackelberg $(k_s^i + k_{BR}(k_s^i))$. The price after the deviation is then the same whichever the deviating firm. In that case, the difference between the initial capacity and the Stackelberg capacity is greater for the smaller firm than for the larger one. The smaller firm then has more incentive to deviate, as it gains more capacity. If both firms are large $(k_0^i \leq k_{BR}(k_s^i)$ for one i) so that the deviation involves prohibiting the other firm from investing, as described by (19), the price after the deviation is higher when the small firm deviates than when the larger firm deviates. Indeed, as the marginal profit of investment decreases with the firm's capacity, the smaller firm has more incentive to invest for the same total capacity in the non-cooperative equilibrium. Therefore, to prohibit the other firm from investing after the deviation, the larger firm has to install a greater capacity than the smaller firm. This price reduction makes the deviation less profitable for the larger firm than for the smaller one. The same reasoning applies when one firm is small and behaves as in (18) and the other one is large and behaves as in (19).

Finally, Lemma 2 and 3 allow us to determine the initial capacities for which there is a possibility of collusion.

Proposition 2 *When $\tau \to 0$, the set of initial capacities such that the agreement for status quo is a Markov perfect equilibrium is:*

$$\Psi = \left\{ k \in \mathbb{R}_+^2 \mid \text{(18) and (19) holds for } i = \arg \inf_{\{A,B\}} \{k_0^i\} \right\}. \tag{20}$$

Furthermore, let k_m be the firms' capacity that maximizes the joint profit,

$$k_m = \arg\max \left\{ \pi_A(k_m, k_m) + \pi_B(k_m, k_m) - 2p^+ k_m \right\} \tag{21}$$

then, if $k_M < k_{Irr}(k_S)$, there is a possibility of collusion if and only if the Stackelberg profit is under half of the joint profit:

$$\pi_i \left(k_s, k_{BR}\left(k_s^i\right)\right) - p^+ k_s \leq \frac{\pi_A(k_m, k_m) + \pi_B(k_m, k_m) - 2p^+ k_m}{2} = \pi_i(k_m, k_m) - p^+ k_m. \tag{22}$$

The parallel between the optimal deviation and the choice of the Stackelberg leader allows us to obtain a clear condition for the possibility of collusion. When half of the monopoly profit is greater than the Stackelberg leader's profit, there is a possibility of collusion. Half of the monopoly profit is the best collusive profit the firms can make and the Stackelberg's leader profit is the deviating profit derived from the monopoly capacity.

In the usual theory of collusion, when firms are more patient (i.e. when their discount rate, δ, increases), the size of the profit at the time of deviation relative to the future profit of punishment decreases, and the collusion is easier to sustain. When investment is irreversible, it is possible to differentiate the condition (18) and (19) with respect to the discount rate to determine its impact on collusion.

Lemma 4 *When $\tau \to 0$, if the discount rate increases, it becomes harder to sustain collusion:*

$$\text{If } \delta' > \delta,\ \Psi_{\delta'} \subset \Psi_{\delta} \tag{23}$$

When firms are more patient, their willingness to invest increases, as they are ready to pay more in the short run to gain profit in the long run. In the case with infinitesimal time periods, deviating from the agreement only impacts firms' long-run profits. Therefore, when firms are more willing to invest, their incentives to deviate increase. This result differs from the usual theory. The reason is that the present case, with no time-to-build, focuses on the long-run profitability of deviation, whereas the usual theory focuses on the short-run profitability of deviation.

General case

This sub-section generalizes the results of the previous section when the time periods are positive ($\tau > 0$). In such case the incentive for deviation is a combination of short-run and long-run effects. Indeed, the deviating firm makes a positive profit during the time between the installation of the deviation capacity and the installation of the punishing capacity. After the installation of the punishing capacity, the deviating firm makes its long-run profit as in the previous sub-section.

In order to state the optimal deviation, let $k_D^i(k_0^j)$ be the capacity that the deviating firm wishes to install if its competitor invests after the deviation, as defined by:

$$(1-\delta^\tau)\frac{\partial \pi_i}{\partial k^i}(k_D^i, k_0^j) + \delta^\tau \left[\frac{\partial \pi_i}{\partial k^i}(k_D^i, k_{BR}\left(k_D^i\right)) + \frac{\partial k_{BR}}{\partial k^i}\left(k_D^i\right)\frac{\partial \pi_i}{\partial k^j}(k_D^i, k_{BR}\left(k_D^i\right))\right] = p^+. \quad (24)$$

This optimal capacity of deviation is below the Stackelberg capacity ($k_D < k_S$).[11] Indeed, the fact that the deviating firm makes a short-run profit reduces its incentives to invest, as the capacity which maximizes the short-run profit of deviation is below that which maximizes the long-run profit. Lemma 5 describes the optimal deviation.

Lemma 5 *The optimal deviation is to install a capacity:*

$$k_d^{*i} = \begin{cases} k_D^i(k_0^j) \text{ if } k_0^j \leq k_{BR}\left(k_D^i\left(k_0^j\right)\right) \\ \left(k_{BR}\right)^{-1}\left(k_0^j\right) \text{ if } k_0^j > k_{BR}\left(k_D^i\left(k_0^j\right)\right) \end{cases}. \quad (25)$$

As in the case with infinitesimal time periods, there are two possible reactions for the deviating firm. If the initial capacity of the punishing firm is low, the punishing firm will invest after the deviation. In such case, the optimal deviation is the maximizing one (24), and firm i accepts the collusive agreement if it makes more profit with its initial capacity than it does by deviating:

If $k_0^j \leq k_{BR}^j\left(k_D^i\left(k_0^j\right)\right)$,

$$\pi_i(k_0) \geq (1-\delta^\tau)\,\pi_i\left(k_D^i\left(k_0^j\right), k_0^j\right) + \delta^\tau \pi_i\left(k_D^i\left(k_0^j\right), k_{BR}\left(k_D^i\left(k_0^j\right)\right)\right) - p^+\left(k_D^i\left(k_0^j\right) - k_0^i\right). \quad (26)$$

When the initial capacity of the punishing firm is sufficiently high, the deviating firm invests less than the capacity defined by (24) in order to take into account the fact that the punishing firm is committed by its initial level of capacity and will not invest after the deviation. Firm i accepts the collusive agreement if it will make more profits is by maintaining its initial capacity than it will by increasing its capacity to the best response level:

If $k_0^j > k_{BR}^j\left(k_D^i\left(k_0^j\right)\right)$,

$$\pi_i(k_0) \geq \pi_i\left(\left(k_{BR}\right)^{-1}\left(k_0^j\right), k_0^j\right) - p^+\left(\left(k_{BR}\right)^{-1}\left(k_0^j\right) - k_0^i\right). \quad (27)$$

[11] The proof of this fact is in appendix.

As previously, the firm with the lowest initial capacity is the one that has the most incentives to deviate from the status quo agreement.

Lemma 6 *The firm with the most incentives to deviate from the collusive agreement is the one with the lowest initial capacity:*

If (26) and (27) holds for i and $k_0^i < k_0^j$, then (26) and (27) holds for firm j.

Lemma 6 allow us to determine the initial capacities for which there is a possibility of collusion.

Proposition 3 *The set of initial capacities such that the agreement for status quo is a Markov perfect equilibrium is:*

$$\Psi = \left\{ k \in \mathbb{R}_+^2 \mid \text{(26) and (27) holds for } i = \arg \inf_{\{A,B\}} \{k_0^i\} \right\}. \tag{28}$$

The condition ensuring that the existence of a possible collusion (meaning that Ψ is not restricted to the Cournot equilibrium) has no general clear mathematical formula. However, when condition (22) is valid, collusion is always possible when τ is small enough.

The impact of the discount factor on the possibility of collusion is ambiguous. Indeed, the two incentives for deviation coexist in the general case. The short-run one, present in standard collusion models, comes from the additional profit made by the deviating firm before its competitor implements punishment. The long-run incentive, previously described in sub-section 3.2, comes from the first mover advantage of the deviating firm due to the irreversibility of investment, and is a direct result of the irreversibility of capacity investment. When the discount factor is very low, firms are impatient, and the short run incentive dominates: an increase of δ augments the possibility of collusion. However, when the discount rate is high, firms are patient, and the long-run incentive prevails. In such case, an increase of δ makes the firms more patient, and willing more to invest and deviate from the collusive agreement in order to obtain a dominant position in the market.

To illustrate this, let H be the possibility of collusion, as defined by:

$$H(k_0) = \pi_i(k_0^i, k_0^j) - \left[(1-\delta^\tau)\,\pi_i(k_D^i, k_0^j) - p^+\left(k_D^i - k_0^i\right) + \delta^\tau \pi_i(k_D^i, k_{BR}\left(k_D^i\right))\right].$$

When $H(k_0) \geq 0$ a possibility for collusion exists, i.e. the grim-trigger strategies for status quo defined by (12) form a Markov perfect equilibrium when $H(k_0) \geq 0$. Figure 3 presents the possibility of collusion in function of the discount factor, in the case of linear demand and quadratic production cost.

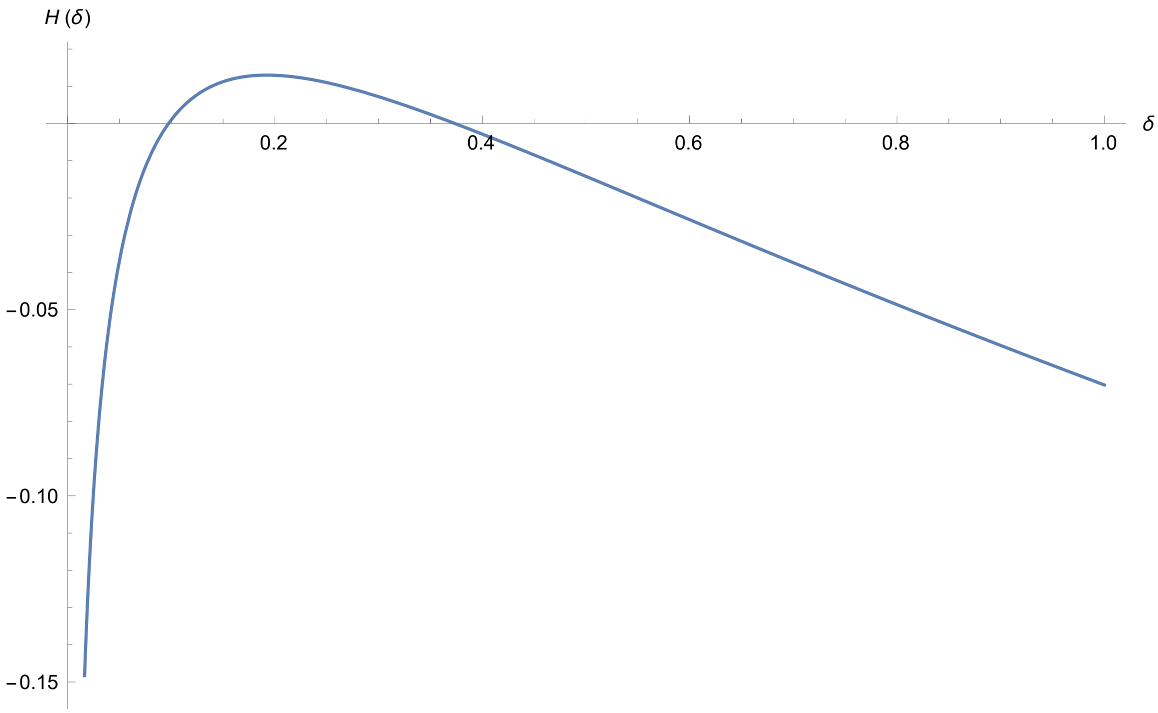

Figure 3: Possibility of collusion

With parameter values: $D(Q) = 1 - Q$, $c(q_i) = 0.1\,(q_i)^2$, $p^+ = 0.3$, $\tau = 0.1$, $k_0^A = k_0^B = 0.15$.

4 Conclusion

This work shows that the irreversibility of investment creates a long-run incentive for the deviation. Indeed, when a firm wishes to deviate from a collusive agreement, it has to increase its production capacity. In doing so, it benefits from a short-run increase in profits before its opponent starts to boost its production as a punishment, but it also commits to this capacity for the rest of the game, due to the irreversibility of investment. This commitment reduces its opponent's incentive to invest in a punishment regime, and permits the deviating firm to gain a long-run advantage in terms of capacity.

This is consistent with the lysine example, in which investment made by the entry firm, ADM, during the first period of collusion (1992-1993), lead to the demise of the cartel. During the ensuing price war, ADM's increased capacity allowed the firm to increase its market share at the expense of its competitors. ADM retained its new market share during the second collusive period that began at the end of 1993.

This work is based two hypotheses, the fact that firms always produce at full capacity, and that industry starts out below capacity (which can be due to an entry, or positive demand shock). These opens up to two different directions for future studies. First, firms can have unused capacity that can be observed by their competitors, and used to enforce cartel agreements. One promising topic for future research would be to determine how this unused capacity facilitates collusion and how it affects firms' investment in capacity. Second, firms often face uncertain market conditions. Market developments may have an impact on capacity collusion, both in relation to the preserving a cartel, and to its creation. An industry dealing with an increase in demand may prefer to collude rather than increase its capacity, in particular if the demand shock is temporary. New research is required to understand the link between cartels in capacity and market evolution.

5 Appendix

Proof of Proposition 1:

The pattern of this proof is the following: first it characterizes a firm's best response, depending on the inter-temporal profit, then it determines this inter-temporal profit at the equilibrium, and finally it gives the equilibrium strategies.

Let $I^j(k)$ be the strategy of firm j. The profit of firm Π_i is then:

$$\Pi_i(k) = \max_{I^i} \left\{ (1-\delta^\tau)\, \pi_i \left(k^i + I^i, k^j + I^j(k) \right) + \delta^\tau \Pi_i \left(k^i + I^i, k^j + I^j(k) \right) - p^+ I^i \right\}. \quad (29)$$

Let $V_i(k)$ be defined by:

$$V_i(k) = (1-\delta^\tau)\,\pi_i\,(k) + \delta^\tau \Pi_i\,(k)\,.$$

Equation (29) can then be rewritten as:

$$\Pi_i(k) = \max_{I^i} \left\{ V_i\left(k^i + I^i, k^j + I^j(k)\,\right) - p^+ I^i \right\}$$

Assuming that V_i is differentiable and concave (in the first variable), the optimal investment of firm i is thus determined by the implicit equation:

$$\frac{\partial V_i}{\partial k_i}\left(k^i + I^i, k^j + I^j(k)\,\right) = p^+. \tag{30}$$

As $\frac{\partial V_i}{\partial k_i}\left(., k^j + I^j(k)\right)$ is decreasing, the best response of firm i is defined by (30) if $\frac{\partial V_i}{\partial k_i}\left(k^i, k^j + I^j(k)\,\right) <$ p^+ and 0 elsewhere.

Therefore, if both firms follow the best response strategy defined previously, no firms invest if $\frac{\partial V_i}{\partial k_i}\left(k^i, k^j\right) \geq p^+$ and $\frac{\partial V_j}{\partial k_i}\left(k^i, k^j\right) \geq p^+$ and, in that case, (k^i, k^j) is a steady point (as investment is irreversible and there is no decrease in capacities, a steady point is defined as a point where no firms invest at the equilibrium). By (29), the inter-temporal profit function at a steady state is given by:

$$\Pi_i\left(k^i, k^j\right) = \pi_i\left(k^i, k^j\right). \tag{31}$$

By (30), firms reach a steady point after their first investment is made. This allow us to rewrite (29), the profit of firm i for a given $I^j(k)$:

$$\Pi_i = (1-\delta^\tau)\,\pi_i\left(k^i + I^i, k^j + I^j(k)\,\right) + \delta^\tau \pi_i\left(k^i + I^i, k^j + I^j(k)\,\right) - p^+ I^i, \tag{32}$$

and gives the equilibrium strategies of the firms: do not invest if

$$\frac{\partial \pi_i}{\partial k_i}\left(k^i, k^j + I^j(k)\right) \geq p^+, \tag{33}$$

and, if not, invest at level I defined by the implicit equation:

$$\frac{\partial \pi_i}{\partial k_i}\left(k^i + I, k^j + I^j(k)\right) = p^+. \tag{34}$$

Note that the above reasoning implies that there is no other Markovian equilibrium such that the inter-temporal profit function (with equilibrium strategies) is differentiable and concave. However, other Markovian equilibria may exists with less regular inter-temporal profits (section 3 shows an example of strategies that are not continuous in the state variables, and that create jumps in the profit function). ■

Lemma 1 is shown in the text. The results of section 3.2, when there is no time-to-build, are the limit case of the results of section 3.3, when there is time-to-build, and therefore are shown after the results of section 3.3.

Proof of footnote number 8:

By assumption A, π is concave, and $\frac{\partial \pi_i}{\partial k^i} < 0$. In particular:

$$\frac{\partial \pi_i}{\partial k^i}(k_D, k_0) < p^+. \tag{35}$$

Therefore:

$$\delta^\tau p^+ < p^+ - (1-\delta^\tau)\frac{\partial \pi_i}{\partial k^i}(k_D, k_0). \tag{36}$$

Equation (24) implies that

$$p^+ - (1-\delta^\tau)\frac{\partial \pi_i}{\partial k^i}(k_D, k_0) = \delta^\tau\left[\frac{\partial \pi_i}{\partial k^i}(k_D, k_{BR}(k_D)) + \frac{\partial k_{BR}}{\partial k^i}(k_D)\frac{\partial \pi_i}{\partial k^j}(k_D, k_{BR}(k_D))\right], \tag{37}$$

and equation (16) implies that

$$\delta^\tau p^+ = \delta^\tau\left[\frac{\partial \pi_i}{\partial k^i}(k_s, k_{BR}(k_s)) + \frac{\partial k_{BR}}{\partial k^i}(k_s)\frac{\partial \pi_i}{\partial k^j}(k_s, k_{BR}(k_s))\right]. \tag{38}$$

Then,

$$\frac{\partial \pi_i}{\partial k^i}(k_s, k_{BR}(k_s)) + \frac{\partial k_{BR}}{\partial k^i}(k_s)\frac{\partial \pi_i}{\partial k^j}(k_s, k_{BR}(k_s)) < \frac{\partial \pi_i}{\partial k^i}(k_D, k_{BR}(k_D)) + \frac{\partial k_{BR}}{\partial k^i}(k_D)\frac{\partial \pi_i}{\partial k^j}(k_D, k_{BR}(k_D)), \tag{39}$$

and, as $\frac{\partial \pi_i}{\partial k^i}(., k_{BR}(.)) + \frac{\partial k_{BR}}{\partial k^i}(.)\frac{\partial \pi_i}{\partial k^j}(., k_{BR}(.))$ is decreasing by assumption H and by (8),

$$k_s > k_D.$$

■

Lemma 5 is the result of the maximization of (14) under the constraint $k_d^i \leq (k_{BR})^{-1}(k_0^j)$, coming from Lemma 1.

Proof of Lemma 6:

Let $\alpha(x)$ be defined by

$$\alpha(x) = \begin{cases} (1-\delta^\tau)\,\pi_i(k_D^i, k_0^j) - p^+\left(k_D^i - k_0^i\right) + \delta^\tau \pi_i\left(k_D^i, k_{BR}\left(k_D^i\right)\right) \text{ if } x \leq k_{BR}\left(k_D^i\left(x\right)\right) \\ \pi_i\left(\left(k_{BR}\right)^{-1}(x), x\right) - p^+\left(k_{BR}\right)^{-1}(x) \text{ if } x > k_{BR}\left(k_D^i\left(x\right)\right) \end{cases}. \tag{40}$$

As the deviation is the optimal one, the derivative of α is (see equation (24) and (8)):

$$\alpha'(x) = \begin{cases} \dfrac{(1-\delta^\tau)\,k_D^i(x)P'(k_D^i(x)+x)}{\ln(1/\delta)} \text{ if } x \leq k_{BR}\left(k_D^i\left(x\right)\right) \\ \dfrac{P'\left(\left(k_{BR}^j\right)^{-1}(x)+x\right)\left(k_{BR}^j\right)^{-1}(x)}{\ln(1/\delta)} \text{ if } x > k_{BR}\left(k_D^i\left(x\right)\right) \end{cases}. \tag{41}$$

With this notation, firm i agrees to collude if and only if:

$$\pi_i(k_0) - p^+k_0^i - \alpha(k_0^j) \geq 0 \tag{42}$$

Let h be a function defined by:

$$h(x) = \frac{P(T)x - c(x)}{\ln(1/\delta)} - p^+x + \alpha(x), \tag{43}$$

where T is a constant which verifies $T < k_D^i(x)+x$ if $x \leq k_{BR}\left(k_{D^*}^i\left(x\right)\right)$ and $T < \left(k_{BR}^j\right)^{-1}(x)+x$ if $x > k_{BR}\left(k_D^i\left(x\right)\right)$. Then, the derivative of h is:

$$h'(x) = \begin{cases} \dfrac{P(T) + (1-\delta^\tau)\,k_{D^*}^i(x)P'(k_D^i(x)+x) - c'(x)}{\ln(1/\delta)} - p^+ \text{ if } x \leq k_{BR}\left(k_D^i\left(x\right)\right) \\ \dfrac{P(T) + P'\left(\left(k_{BR}\right)^{-1}(x)+x\right)\left(k_{BR}\right)^{-1}(x) - c'(x)}{\ln(1/\delta)} - p^+ \text{ if } x > k_{BR}\left(k_D^i\left(x\right)\right) \end{cases}. \tag{44}$$

As $P(T) > P(k_D^i(x)+x)$ if $x \leq k_{BR}\left(k_D^i\left(x\right)\right)$ and $P(T) > \left(k_{BR}\right)^{-1}(x)+x$ if $x > k_{BR}\left(k_D^i\left(x\right)\right)$, the derivative of h is positive. Then, if $k_0^j > k_0^i$,

$$\frac{P(T)k_0^j - c(k_0^j)}{\ln(1/\delta)} - p^+k_0^j + \alpha(k_0^j) \geq \frac{P(T)k_0^i - c(k_0^i)}{\ln(1/\delta)} - p^+k_0^i + \alpha(k_0^i) \tag{45}$$

By assuming that $T = k_0^i + k_0^j$, (45) can be rewritten:

$$\pi_j(k_0) - p^+k_0^j - \alpha(k_0^i) \geq \pi_i(k_0) - p^+k_0^i - \alpha(k_0^j). \tag{46}$$

Therefore, if $k_0^j > k_0^i$, and firm i agrees for collusion, then firm j also agrees, as:

$$\pi_i(k_0) - p^+k_0^i - \alpha(k_0^j) \geq 0 \Rightarrow \pi_j(k_0) - p^+k_0^j - \alpha(k_0^i). \tag{47}$$

■

Proposition 3 comes from the combination of Lemmas 5 and 6. In section 3.2, Lemma 2 is a corollary of Lemma 5 and Lemma 3 is a corollary of Lemma 6.

Proof of Proposition 2:

The first part of proposition 2 comes from the combination of Lemmas 2 and 3.

To state the second part of proposition 2, note that the incentive for collusion of firm i (IC_{coll}) can be written:

$$IC_{\text{coll}} = \begin{cases} \pi_i(k_0) - p^+ k_0^i - \left(\pi_i(k_s, k_{BR}(k_s)) - p^+ k_s^i\right) \text{ if } k_0^j \leq k_{Irr}(k_s) \\ \pi_i(k_0) - p^+ k_0^i - \left(\pi_i\left((k_{BR})^{-1}(k_0^j), k_0^j\right) - p^+ k_{BR}^{-1}(k_0^j)\right) \text{ if } k_0^j > k_{BR}(k_s) \end{cases}.$$

For a given initial state, the collusive agreement for the status quo is a Markovian equilibrium if, and only if, this incentive for collusion is positive. To show wether or not there is a possibility for collusion, I determine the initial state maximizing the incentive for collusion and study the sign of IC_{coll} in this particular state.

As the smallest firm has a greater incentive to deviate than its opponent, the initial state maximizing IC_{coll} is symmetric. In the following, k_0 designs alternatively the capacity of one of the firms or the vector of capacity, (k_0, k_0).

First, assume that $k_0 > k_{Irr}(k_S)$. Then, the incentive to collude is:

$$IC_{\text{coll}} = \pi_i(k_0, k_0) - p^+ k_0 - \left(\pi_i\left((k_{BR})^{-1}(k_0), k_0\right) - p^+ (k_{BR})^{-1}(k_0)\right). \tag{48}$$

The second term, $\pi_i\left((k_{BR})^{-1}(k_0), k_0\right) - p^+ (k_{BR})^{-1}(k_0)$ is the value of the maximization of $\pi_i(k_s^i, k_{BR}(k_s^i)) - p^+ k_s^i$ in the boundary $(k_{BR})^{-1}(k_0)$. Therefore, its differential is positive (as $k_{BR}^{-1}(k_0) < k_S$). The first term is maximized in $k_m < k_{BR}(k_S^i)$, as the profit is symmetric and:

$$k_m = \arg\max\left\{\pi_A(k_m, k_m) + \pi_B(k_m, k_m) - 2p^+ k_m\right\} = \arg\max\left\{\pi_i(k_m, k_m) - p^+ k_m\right\}. \tag{49}$$

Therefore, the differential of $\pi_i(k_0, k_0) - p^+ k_0$ is negative, as for the differential of IC_{coll}, and the initial state maximizing IC_{coll} should be below $k_{BR}(k_s)$.

Now, assume that $k_0 < k_{BR}(k_s)$. Then, the incentive to collude is:

$$IC_{\text{coll}} = \pi_i(k_0) - p^+ k_0^i - \left(\pi_i(k_s, k_{BR}(k_s)) - p^+ k_s^i\right),$$

which is maximized in k_m.

$\blacksquare$

Proof of lemma 4:

The pattern of proof is to fix a vector of initial capacity such that firm i has an incentive to deviate, and to show that firm i has an incentive to deviate for a higher δ. The proof is decomposed into two parts, depending whether $k_0^j \leq k_{BR}\left(k_s^i\right)$ or not.

Before doing so, note that the best response of capacity, $k_{BR}(x)$, is an increasing function function of the discount rate, δ (for all $x \in \mathbb{R}_+$). Indeed, the implicit equation defining k_{BR}, (8), can be rewritten:

$$P'\left(x + k_{BR}\left(x\right)\right) k_{BR}\left(x\right) + P\left(x + k_{BR}\left(x\right)\right) - c'\left(k_{BR}\left(x\right)\right) = \ln\left(\frac{1}{\delta}\right) p^+. \tag{50}$$

This gives, by differentiation:

$$\frac{\partial k_{BR}\left(x\right)}{\partial \delta} = -\frac{1}{\delta}\frac{p^+}{\left[P''\left(x + k_{BR}\left(x\right)\right) k_{BR}\left(x\right) + 2P'\left(x + k_{BR}\left(x\right)\right) - c''\left(k_{BR}\left(x\right)\right)\right]}. \tag{51}$$

By assumption A, the second derivative of the firm's payoff is negative, and therefore $\frac{\partial k_{BR}(x)}{\partial \delta} > 0$, meaning that a more patient firm invest a higher quantity.

Assume that $k_0^j \leq k_{BR}\left(k_s^i\right)$. Then, firm i has an incentive to deviate if and only if:

$$\pi_i(k_0) - p^+ k_0^i < \pi_i(k_s, k_{BR}\left(k_s\right)) - p^+ k_s^i, \tag{52}$$

which is equivalent to

$$P\left(k_0^i + k_0^{-i}\right) k_0^i - c(k_0^i) < P\left(k_s + k_{BR}\left(k_s\right)\right) k_0^i - c(k_s^i) - \ln\left(\frac{1}{\delta}\right) p^+ \left(k_s - k_0^i\right). \tag{53}$$

By definition of k_s this can be rewritten,

$$P\left(k_0^i + k_0^{-i}\right) k_0^i - c(k_0^i) < \max_x \left\{P\left(x + k_{BR}\left(x\right)\right) x - c(x) - \ln\left(\frac{1}{\delta}\right) p^+ \left(x - k_0^i\right)\right\}. \tag{54}$$

The first term of the inequality does not depend on δ. To show that an increase of δ augments the incentive to deviate, it is enough to show that the derivative of the second term is positive. Let $V(\delta) = \max_{x \in \mathbb{R}_+} \left\{P\left(x + k_{BR}\left(x\right)\right) x - c(x) - \ln\left(\frac{1}{\delta}\right) p^+ \left(x - k_0^i\right)\right\}$. Then, the envelope theorem implies that:

$$V'(\delta) = \frac{\partial k_{BR}\left(k_S\right)}{\partial \delta} P'\left(k_s + k_{BR}\left(k_s\right)\right) k_s + \frac{1}{\delta} p^+ \left(k_s - k_0^i\right), \tag{55}$$

by using (54),

$$V'(\delta) = \frac{p^+}{\delta}\left(\left(k_s - k_0^i\right) - \frac{P'\left(k_s + k_{BR}\left(k_s\right)\right)k_s}{P''\left(k_s + k_{BR}\left(k_s\right)\right)k_{BR}\left(k_s\right) + 2P'\left(k_s + k_{BR}\left(k_s\right)\right) - c''\left(k_{BR}\left(k_s\right)\right)}\right), \tag{56}$$

thus

$$V'(\delta) = \frac{p^+}{\delta}k_s\left(\frac{P''\left(k_s + k_{BR}\left(k_s\right)\right)k_{BR}\left(k_s\right) + P'\left(k_s + k_{BR}\left(k_s\right)\right) - c''\left(k_{BR}\left(k_s\right)\right)}{P''\left(k_s + k_{BR}\left(k_s\right)\right)k_{BR}\left(k_s\right) + 2P'\left(k_s + k_{BR}\left(k_s\right)\right) - c''\left(k_{BR}\left(k_s\right)\right)}\right) - \frac{p^+}{\delta}k_0^i. \tag{57}$$

As $P' < 0$,

$$\frac{P''\left(k_s + k_{BR}\left(k_s\right)\right)k_{BR}\left(k_s\right) + P'\left(k_s + k_{BR}\left(k_s\right)\right) - c''\left(k_{BR}\left(k_s\right)\right)}{P''\left(k_s + k_{BR}\left(k_s\right)\right)k_{BR}\left(k_s\right) + 2P'\left(k_s + k_{BR}\left(k_s\right)\right) - c''\left(k_{BR}\left(k_s\right)\right)} > 1, \tag{58}$$

and $V'(\delta) > 0$ as $k_s > k_0^i$ by assumption ($k_0^i < k_c$).

Assume now that $k_0^j > k_{BR}\left(k_s^i\right)$. To simplify the notation, let $k_{BR}^{-1} = \left(k_{BR}\right)^{-1}\left(k_0^j\right)$. Then, firm i has an incentive to deviate if and only if:

$$P\left(k_0^i + k_0^{-i}\right)k_0^i - c(k_0^i) < P\left(k_{BR}^{-1} + k_0^j\right)\right)k_{BR}^{-1} - c\left(k_{BR}^{-1}\right) - \ln\left(\frac{1}{\delta}\right)p^+\left(k_{BR}^{-1} - k_0^i\right). \tag{59}$$

Let $V(\delta) = P\left(k_{BR}^{-1} + k_0^j\right)k_{BR}^{-1} - c\left(k_{BR}^{-1}\right) - \ln\left(\frac{1}{\delta}\right)p^+\left(k_{BR}^{-1} - k_0^i\right)$. The derivative of the second term is given by:

$$V'(\delta) = \frac{\partial k_{BR}^{-1}}{\partial \delta}\left(P'\left(k_{BR}^{-1} + k_0^j\right)k_{BR}^{-1} + P\left(k_{BR}^{-1} + k_0^j\right) - c'\left(k_{BR}^{-1}\right) - \ln\left(\frac{1}{\delta}\right)p^+\right) + \frac{1}{\delta}p^+\left(k_{BR}^{-1} - k_0^i\right). \tag{60}$$

As $k_{BR}^{-1} < k_s$, $P'\left(k_{BR}^{-1} + k_0^j\right)k_{BR}^{-1} + P\left(k_{BR}^{-1} + k_0^j\right) - c'\left(k_{BR}^{-1}\right) - \ln\left(\frac{1}{\delta}\right)p^+$ is positive. Furthermore, as k_{BR} increases with δ, k_{BR}^{-1} also increases with δ and thus $V'(\delta) > 0$.

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